NAG Toolbox for MATLAB

f04ax

1 Purpose

f04ax calculates the approximate solution of a set of real sparse linear equations with a single right-hand side, $Ax = b$ or $A^{\mathrm{T}}x = b$, where A has been factorized by f01br or f01bs.

2 Syntax

```
[rhs, resid] = f04ax(a, icn, ikeep, rhs, mtype, idisp, 'n', n, 'licn', licn)
```

3 Description

To solve a system of real linear equations $Ax = b$ or $A^{\mathrm{T}}x = b$, where A is a general sparse matrix, A must first be factorized by f01br or f01bs. f04ax then computes x by block forward or backward substitution using simple forward and backward substitution within each diagonal block.

The method is fully described in Duff 1977.

A more recent method is available through solver function f11mf and factorization function f11me .

4 References

Duff I S 1977 MA28 – a set of Fortran subroutines for sparse unsymmetric linear equations *AERE Report R8730* HMSO

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**licn**) – **double array**

The nonzero elements in the factorization of the matrix A, as returned by f01br or f01bs.

2: **icn**(**licn**) – **int32 array**

The column indices of the nonzero elements of the factorization, as returned by f01br or f01bs.

3: **ikeep**(**5** × **n**) – **int32 array**

ikeep provides, on entry, indexing information about the factorization, as returned by f01br or f01bs. Used as internal workspace prior to being restored and hence is unchanged.

4: **rhs**(**n**) – **double array**

The right-hand side vector b.

5: **mtype** – **int32 scalar**

mtype specifies the task to be performed.

mtype $= 1$

Solve $Ax = b$.

mtype $\neq 1$

Solve $A^{\mathrm{T}}x = b$.

6: **idisp**(2) **– int32 array**

The values returned in **idisp** by f01br.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **rhs**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **licn – int32 scalar**

Default: The dimension of the arrays **a**, **icn**. (An error is raised if these dimensions are not equal.)

5.3 Input Parameters Omitted from the MATLAB Interface

w

5.4 Output Parameters

1: **rhs**$(\mathbf{n})$ **– double array**

The array contains the solution vector x.

2: **resid – double scalar**

The value of the maximum residual, $\max\left(\left|b_i - \sum_j a_{ij}x_j\right|\right)$, over all the unsatisfied equations, in case f01br or f01bs has been used to factorize a singular or rectangular matrix.

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{n} < 0$.

7 Accuracy

The accuracy of the computed solution depends on the conditioning of the original matrix. Since f04ax is always used with either f01br or f01bs, you are recommended to set GROW = **true** on entry to these functions and to examine the value of $\mathbf{w}(1)$ on exit (see f01br and f01bs). For a detailed error analysis see page 17 of Duff 1977.

If storage for the original matrix is available then the error can be estimated by calculating the residual

$$r = b - Ax \qquad \left(\text{or } b - A^{\mathrm{T}}x\right)$$

and calling f04ax again to find a correction δ for x by solving

$$A\delta = r \qquad \left(\text{or } A^{\mathrm{T}}\delta = r\right).$$

8 Further Comments

If the factorized form contains τ nonzeros (**idisp**$(2) = \tau$) then the time taken is very approximately 2τ times longer than the inner loop of full matrix code. Some advantage is taken of zeros in the right-hand side when solving $A^{\mathrm{T}}x = b$ (**mtype** $\neq 1$).

9 Example

```
n = int32(6);
nz = int32(15);
a = zeros(150,1);
a(1:15) = [5; 2; -1; 2; 3; -2; 1; 1; -1; -1; 2; -3; -1; -1; 6];
irn = zeros(75,1,'int32');
irn(1:15) = [int32(1); int32(2); int32(2); int32(2); int32(3); int32(4);
...
              int32(4); int32(4); int32(5); int32(5); int32(5); int32(5);
...
             int32(6); int32(6); int32(6)];
icn = zeros(150,1,'int32');
icn(1:15) = [int32(1); int32(2); int32(3); int32(4); int32(3); int32(1);
...
              int32(4); int32(5); int32(1); int32(4); int32(5); int32(6);
...
             int32(1); int32(2); int32(6)];
abort = [true;
     true;
     false;
     true];

rhs = [15;
     12;
     18;
     3;
     -6;
     0];
mtype = int32(1);
[a, irn, icn, ikeep, w, idisp, ifail] = f01br(n, nz, a, irn, icn, abort);
[rhsOut, resid] = f04ax(a, icn, ikeep, rhs, mtype, idisp)
```

```
rhsOut =
     3
     3
     6
     6
     3
     1
resid =
     0
```